The Diurnal Variation of Terrestrial Magnetism.

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The diurnal variation of terrestrial magnetism has been the subject of experimental study for many years, and at a considerable number of observatories scattered all over the earth. But the co-ordination of the results, and the theoretical investigation of the physical cause or causes of the phenomena, have not made progress which seems to bear a reasonable proportion to the vast amount of observational data that has accumulated.

As far as I know, Dr. Arthur Schuster's memoirs* constitute the most inspiring and systematic attempt to reduce this matter to scientific law and order. Although I have found it convenient to depart from the method pursued by Schuster, this change of procedure was in no small degree suggested by various remarks in his memoirs.

The comparative lack of interest in the subject arises, I believe, in great measure from the difficulty (commented on by Schuster) of obtaining the experimental data in a form convenient for rational comparison. Observatories still continue to reduce their observations and to publish their results in a variety of ways, and, unless one is prepared to undertake a considerable amount of tedious arithmetical computation at the very outset, it is practically impossible to obtain a comprehensive view of what the facts are.

The Advisory Committee of Eskdalemuir Observatory recommended that the instruments should be arranged so as to record directly the geographical components of magnetic force, and, when the tabulated results for 1911 began to take definite shape, it was, perhaps, natural that I should be interested in comparing the results with those of the only other observatory which at the time also recorded the geographical components, namely, Potsdam (Seddin). The comparison, which I made in the light of Schuster's conclusions, brought out points of such interest that I was induced to attempt to collect data from other observatories as to the deduced values for the geographical components. Following Schuster, the data I desired to obtain were the Fourier coefficients in the geographical components. I confined myself to the 24-hour and the 12-hour terms, because I share the view, entertained by many of those who realise the errors that arise on the experimental and observational side, that the shorter period terms are of very doubtful accuracy.

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* 'Phil. Trans.,' 1899, vol. 180, and 1908, vol. 208.

I further confined my attention to the average yearly value, not because the seasonal variation is unimportant, but because I felt that there was a great danger of coming to grief in a multitude of facts.

The data I have been able to obtain are collected in tabular form. If they appear somewhat meagre, I would point out that I had perforce to confine my attention to observatories whose results could, without very much arithmetical labour, be put in the form desired. I hope that the information disclosed by the table will encourage others to make a contribution to it, and thus to the progress of knowledge of the phenomena.

In the table the quantities are those in the equivalent formulæ

$$a_1 \cos t + b_1 \sin t + a_2 \cos 2t + b_2 \sin 2t$$

and

$$c_1 \sin (t + \alpha_1) + c_2 \sin (2t + \alpha_2),$$

where t is the local mean time at the station and the unit of magnitude is $0{\cdot}1\,\gamma$ or 10^{-6} gauss.

From information very kindly supplied by the directors of the various observatories, the following statement may be made:—

1. Practically all days are used, not selected quiet days.
2. The original curves are not smoothed, except in the case of the V results for Pola.
3. The original curves gave records of H and D except at Eskdalemuir and Seddin, where N and W were directly recorded.
4. The hourly values are those at the exact hour, except at Seddin, where the hourly value is the estimated mean for an hour centering at the exact hour.

I need perhaps hardly say that one could wish for data obtained in precisely the same way, and that before entering on a minute arithmetical computation, all data should refer to the same year. It was soon obvious from my inquiries that one would have to wait several years before such data could be obtained.

Nevertheless, I think certain broad inferences may be drawn from the data collected in the table, and these seem to me of vital importance.

It will be convenient to review briefly the main points in Schuster's memoirs. If we grant the main proposition, that it is correct to represent the variations by a potential function, the problem may be divided into two parts:—

(1) The empirical determination of the potential function which represents the observations.

Table I.—Fourier Coefficients of the 24-hour and 12-hour Terms in the Geographical Components of Diurnal Variation of Terrestrial Variation.

Station.		Pavlovsk.	Eskdalemuir.	Wilhelmshaven.	Potsdam.		De Bilt.	Pola.		Helwan.	Bombay.	Batavia.
Lat.		59 ·7° N.	55 ·3° N.	53 ·5° N.	52 ·3° N.		52 ·1° N.	44 ·9° N.		29 ·9° N.	18 ·9° N.	6 ·2° S.
Long.		30 ·5° E.	3 ·2° W.	8 ·2° E.	13 ·0° E.		5 ·2° E.	13 ·9° E.		31 ·4° E.	72 ·8° E.	106 ·8° E.
Year.		1907.	1911.	1910.	1911.	1910.	1910.	1911.	1900 to 1909.	1909.	1904.	1908.
North.	a_1	103	107	105	96	112	118	53	60	− 63	−161	−169
	b_1	− 30	− 38	− 13	− 10	− 12	− 16	16	21	25	40	68
	c_1	107	114	105	97	113	119	56	63	− 68	−166	−182
	α_1	106°	110°	97°	96°	96°	98°	73°	71°	112°	104°	112°
	a_2	− 66	− 68	− 52	− 37	− 47	− 55	0	− 20	43	86	69
	b_2	13	8	1	− 3	3	2	− 4	5	25	− 15	− 24
	c_2	67	68	52	37	47	55	4	21	− 50	− 87	− 73
	α_2	281°	276°	271°	266°	274°	268°	179°	284°	240°	280°	290°
West.	a_1	− 85	− 76	− 97	− 77	− 93	− 96	− 30	− 92	− 66	− 22	− 9
	b_1	− 95	− 64	− 60	− 54	− 65	− 68	− 53	− 78	− 75	− 34	78
	c_1	128	99	114	95	113	118	88	121	100	41	79
	α_1	222°	230°	238°	236°	235°	235°	233°	230°	221°	213°	354°
	a_2	15	34	50	47	48	51	49	48	46	51	23
	b_2	31	64	66	66	74	70	75	92	67	27	− 84
	c_2	34	72	82	81	88	87	90	104	81	58	87
	α_2	25°	28°	37°	35°	33°	36°	33°	28°	35°	62°	165°
Vertical (down).	a_1	− 16	− 4		16	19	20	− 8	51	54	52	−140
	b_1	− 68	− 71		− 36	− 42	− 49	− 14	− 18	− 14	2	40
	c_1	70	71		39	46	53	16	54	56	52	−145
	α_1	193°	183°		156°	156°	158°	210°	110°	105°	88°	106°
	a_2	− 34	− 42		− 36	− 38	− 36	− 46	− 42	− 43	− 38	75
	b_2	− 15	− 5		− 2	− 2	− 6	13	2	17	28	43
	c_2	37	42		36	38	37	48	42	46	47	− 86
	α_2	246°	263°		266°	267°	261°	286°	273°	292°	306°	240°

(2) The theoretical investigation of the physical causes that give the potential function so determined.

These two divisions are substantially represented in Schuster's first and second memoirs. But the two divisions are not entirely independent, (1) must borrow from any hint suggested in (2), and (2) must be kept within the bounds indicated by (1).

In the first memoir, Schuster dealt with observations from the four observatories—Pavlovsk, Greenwich, Lisbon and Bombay. Observing that in the west component the Fourier terms depended substantially on the local time (a feature which is in the main confirmed by our table), he showed how the potential could be calculated. Since the west component is of the form $\delta\Omega/\sin\theta\delta\phi$, where θ is the co-latitude and ϕ the longitude, it is only necessary to get the proper expression for W in Tesseral harmonics, and then the simple integration with respect to ϕ gives, when multiplied by $\sin\theta$, the function Ω.

His conclusion reached was that the principal parts of the mean diurnal variation for the year could be expressed by a potential function of the form

$$\Omega/a = \mathrm{A}_1 \sin\theta\cos\theta\cos(t+\alpha_1) + \mathrm{A}_2\sin^2\theta\cos\theta\cos(2t+\alpha_2),$$

where t is the local time or the equivalent Greenwich time plus the longitude, say $(t'+\phi)$.

In the second memoir Schuster proposes to explain these terms as arising from electrical currents in the atmosphere, set up by the joint action of the permanent part of the earth's magnetic field and the mechanical oscillation of the atmosphere. Thus the barometric variation is associated with the magnetic diurnal variation. Certain formidable difficulties occur and are carefully noted by Schuster, but I need not comment on them.

In attempting to deal with the seasonal variation Schuster supposes that the conductivity of the air depends on the sun's zenith distance, and, assuming a simple and tentative expression for the conductivity, investigates the potential function that arises. The analysis is necessarily exceedingly complicated.

Let us now return to the empirical expression obtained by Schuster.

A potential of the form

$$\Omega/a = \mathrm{A}_1\sin\theta\cos\theta\cos(t'+\phi+\alpha_1)$$

must give components to north N and to west W

$$\mathrm{N} = +\mathrm{A}_1\cos 2\theta\cos(t'+\phi+\alpha_1), \qquad \mathrm{W} = -\mathrm{A}_1\cos\theta\sin(t'+\phi+\alpha_1);$$

and again the form

$$\Omega/a = \mathrm{A}_2\sin^2\theta\cos\theta\cos(2t'+2\phi+\alpha_2)$$

gives components

$$N = +A_2 \sin\theta\,(2-3\sin^2\theta)\cos(2t'+2\phi+\alpha_2),$$

$$W = -A_2 \sin 2\theta \sin(2t'+2\phi+\alpha_2).$$

The following Table II facilitates the comparison of the formulæ just obtained with the data collected in Table I:—

Table II.

Station.	Sin θ.	Cos θ.	Cos 2θ.	Sin 2θ.	Sin $\theta\,(2-3\sin^2\theta)$.
Pavlovsk	0·505	0·863	0·490	0·871	0·624
Eskdalemuir	0·569	0·822	0·352	0·936	0·586
Wilhelmshaven	0·594	0·804	0·293	0·956	0·560
Potsdam	0·612	0·791	0·252	0·968	0·536
De Bilt	0·614	0·789	0·245	0·969	0·534
Pola	0·709	0·705	−0·005	1·000	0·349
Helwan	0·867	0·498	−0·504	0·864	−0·221
Bombay	0·946	0·328	−0·790	0·614	−0·648
Batavia	0·994	−0·108	−0·977	−0·214	−0·958

If now we examine the data and consider W only, then, making allowance for the fact that the data are not all for the same year, we have, I think, strong support for Schuster's expression. Batavia is, however, abnormal.

Again, taking the values for N only we have strong support for a formula of the type given by Schuster.

But clearly for any one station the values of N and W should give the same constants in the formula. This is not the case. The phase angles are not in agreement, and what is even more serious is that the observed amplitudes of the north component c_1 and c_2 are all too great as compared with the values computed from the potential function which represents the west component. The matter looks still more difficult if we compare the a_1, b_1, a_2, b_2 of N with the b_1, a_1, b_2 and a_2 of W.

I confess that the result was disconcerting, for one could see no flaw in Schuster's method, and the west values did, on the whole, depend mainly on local time. There was, of course, the possibility of a higher zonal harmonic, but this did not promise much help, and I was certainly unwilling to enter on much arithmetical computation. There remained the possibility of obtaining a function which would contribute to N without contributing largely to W. This practically meant a term depending on time from some fixed meridian, as Schuster had indicated.

After carefully studying Schuster's theory of the cause, with its ensuing complicated analysis, it appeared to me that one might reverse the whole process, that is to say, one may in a perfectly general way determine potential functions which are differentially related to opposite sides of the earth. The process may stop as soon as we get the terms which represent the data, and then we may seek for the physical meaning of the terms. In mathematical terms we start with the primary potential $1/r$. Any space derivative of this is also a potential. Thus a space derivative fixed with regard to the earth gives a contribution to the fixed part of the earth's field, while a space derivative fixed with regard to the sun gives a contribution to diurnal change.

In this simple way expressions were formed and it was found that the terms sought for could arise. I think it will be convenient to postpone consideration of the manner in which they were obtained and to give first the results.

For the 24-hour term the form of potential function at which I arrived may be written

$$\Omega = \mathrm{A} \sin\theta \cos\theta \cos(t+\alpha_1) + \mathrm{B} \sin\theta \cos(t+\alpha_2) \\ + \mathrm{C}(\sin^2\theta - \tfrac{2}{3}) \cos(t+\phi'-\phi) + \mathrm{D} \sin^2\theta \cos(t+\phi-\phi'),$$

wherein t is the local mean time $(= t'+\phi)$ and θ, ϕ are the co-latitude and longitude of the station, while ϕ' may be regarded as the longitude of some arbitrary meridian.

Since the data do not all refer to the same year a minute numerical analysis would be out of place. Thus only round numbers were selected for the constants. By trial the following form was finally selected :—

$$10^6\,\Omega/a = (150 \sin\theta \cos\theta - 50 \sin\theta) \cos t + (-70 \sin\theta \cos\theta - 10 \sin\theta) \sin t \\ + 65 (\sin^2\theta - \tfrac{2}{3}) \cos(t-\phi) + 5 \sin^2\theta \cos(t+\phi).$$

This gives in units of $0{\cdot}1\,\gamma$ the following values for the north and west components :—

$$\mathrm{N} = (150 \cos 2\theta - 50 \cos\theta + 70 \sin^2\theta \cos\phi) \cos t \\ + (-70 \cos 2\theta - 10 \cos\theta + 60 \sin^2\theta \sin\phi) \sin t,$$

$$\mathrm{W} = (-70 \cos\theta - 10 - 10 \sin\theta \sin\phi) \cos t \\ + (-150 \cos\theta + 50 - 10 \sin\theta \cos\phi) \sin t.$$

The numerical values computed are shown in Table III.

Although the observed values are given in units of $0{\cdot}1\,\gamma$, it must be remembered that the accuracy in most cases does not exceed $1\,\gamma$.

I think it will be admitted that we have, on the whole, got a substantial representation of the data. The better agreement that might have been obtained for the European stations had, of course, to be sacrificed somewhat to get the curious features exhibited by the Bombay and Batavia data.

Table III.

Station.	N.		W.		V. (Provisional.)	
	a_1.	b_1.	a_1.	b_1.	a_1.	b_1.
Pavlovsk	+ 83	−16	−73	−84	+18	−50
Eskdalemuir	+ 77	−36	−67	−79	+20	−35
Wilhelmshaven	+ 70	−21	−67	−76	+24	−42
Potsdam	+ 64	−13	−67	−75	+26	−45
De Bilt	+ 65	−20	−66	−74	+25	−42
Pola	+ 32	+ 7	−61	−63	+32	−45
Helwan	− 49	+57	−49	−32	+29	−38
Bombay	−142	+87	−42	− 2	+ 4	−21
Batavia	−137	+59	−12	+69	−74	+13

The potential function obtained must also account for the values of V. We are at liberty to suppose that any term in Ω arises from an external or interior source, and further that these contributions may differ in phase. We may, therefore, introduce a division of the constants that will give the correct value of Ω at the surface and at the same time account for the observed values of V. In fact, if we could depend on the values of V we have the important means of determining the external and internal proportions. But unfortunately the observations of V are poor and everyone who has really faced the experimental difficulties admits that the results are most unsatisfactory. In particular I know that the Eskdalemuir results for V are quite unreliable, and, judging from the other data, I think they must be regarded with some suspicion. The Potsdam and De Bilt results do, however, agree very well, and so I take them as a sort of standard.

But clearly the position is a weak one, and therefore I do not feel justified in giving more than a general indication of what our formula would do.

I therefore assume that the outside and inside contributions agree in phase, and as regards the second order spherical harmonics I adopt Schuster's result that the internal contribution is one quarter of the external. In the first order harmonic I assume that the contribution from internal source is nil.

We thus get the formula for V.

$$\begin{aligned} V = {} & \{150 \sin\theta \cos\theta - 50 \sin\theta + (70 \sin^2\theta - 130/3) \cos\phi\} \cos t \\ & + \{-70 \sin\theta \cos\theta - 10 \sin\theta + (60 \sin^2\theta - 130/3) \sin\phi\} \sin t. \end{aligned}$$

The computed values, marked provisional, are entered in Table III. The results are, on the whole, in the right direction, and might be brought closer by taking a smaller proportion from an internal source, but I do not think it worth while to force the matter until better data are available.

We now consider the 12-hour terms. The form which I found it convenient to try was

$$\Omega = \text{A} \sin^2\theta \cos\theta \cos(2t+\alpha_1) + \text{B} \sin^2\theta \cos(2t+\alpha_2)$$
$$+\tfrac{1}{3}\text{C} \sin^3\theta \sin(2t+\phi-\phi') + \text{D} \sin\theta\,(\tfrac{4}{5}-\sin^2\theta) \sin(2t-\phi+\phi')$$
$$+\text{E} \sin\theta \sin(2t-\phi+\phi').$$

The first four terms involve harmonics of order 3 but the last is of order 1.

I finally dropped the term in B, made $\text{E} = -\frac{4}{5}\text{D}$ and $\text{D} = \frac{1}{3}\text{C}$. It also seemed an advantage to make $\phi' = 30°$.

Thus the empirical form adopted was

$$10^6\,\Omega/a = \{-60 \sin^2\theta \cos\theta + 40 \sin^3\theta \sin(\phi-30°)\} \cos 2t$$
$$+25 \sin^2\theta \cos\theta \sin 2t,$$

so that $\text{C} = 60, \quad \text{D} = 20, \quad \text{E} = -16.$

The components to north and west thus become

$$\text{N} = \{-60 \sin\theta\,(2-3\sin^2\theta) + 120 \sin^2\theta \cos\theta \sin(\phi-30°)\} \cos 2t$$
$$+25 \sin\theta\,(2-3\sin^2\theta) \sin 2t,$$
$$\text{W} = \{60 \sin 2\theta - 80 \sin^2\theta \sin(\phi-30°)\} \sin 2t$$
$$+\{25 \sin 2\theta + 40 \sin^2\theta \cos(\phi-30°)\} \cos 2t.$$

The computed values are shown in the following Table IV :—

Table IV.

Station.	N.		W.		V. (Provisional.)	
	a_2.	b_2.	a_2.	b_2.	a_2.	b_2.
Pavlovsk	−37	+16	+32	+52	−34	+ 8
Eskdalemuir	−50	+15	+35	+68	−44	+11
Wilhelmshaven	−47	+14	+37	+68	−45	+ 9
Potsdam	−43	+13	+39	+67	−47	+10
De Bilt	−47	+13	+38	+71	−49	+ 9
Pola	−32	+ 9	+44	+71	−58	+10
Helwan	+12	− 6	+52	+51	−59	+14
Bombay	+63	−16	+42	−12	−10	+24
Batavia	+45	−24	+ 3	−89	+69	+29

Again, I think these numbers give substantial agreement with the data.

Turning to the data for V, the numbers look more hopeful than in the case of the 24-hour terms, and the experimental errors that affect the 24-hour term are perhaps not so serious in the 12-hour term. Thus I was tempted to push the agreement somewhat further.

The data suggested that the main part for the northern stations arose from the term in $\sin^2\theta\cos\theta$, but was chiefly in the cosine term $\cos 2t$ and practically nil for the sine term $\sin 2t$. This can be met by assuming a phase difference between the outside and inside contributions. Thus I assumed

$$10^6\,\Omega/a = \mathrm{A}\frac{r^3}{a^3}\sin^2\theta\cos\theta\cos(2t-\epsilon_1)+\mathrm{B}\frac{a^4}{r^4}\sin^2\theta\cos\theta\cos(2t-\epsilon_2).$$

Hence we have

$$\mathrm{A}\cos\epsilon_1+\mathrm{B}.\cos\epsilon_1 = -60, \qquad \mathrm{A}\sin\epsilon_1+\mathrm{B}\sin\epsilon_2 = 25.$$

To get the V results for Potsdam, we have the additional equations

$$3\mathrm{A}\cos\epsilon_1-4\mathrm{B}\cos\epsilon_2 = -120, \qquad 3\mathrm{A}\sin\epsilon_1-4\mathrm{B}\sin\epsilon_2 = 0.$$

Hence $\mathrm{A} = -53$, $\mathrm{B} = -14$, $\epsilon_1 = -15°$, $\epsilon_2 = -50°$, which agrees with Schuster's result that B is about 1/4 of A.

Next the term in $\sin^3\theta$ suggested the main part of the terms at Batavia, and assuming for the moment that the harmonic was entirely of order 3, I took the form

$$10^6\,\Omega/a = \sin^3\theta\sin(\phi-30)\left\{\mathrm{A}\frac{r^3}{a^3}\cos(2t-\epsilon_1)+\mathrm{B}\frac{a^4}{r^4}\cos(2t-\epsilon_2)\right\}.$$

Hence $\quad \mathrm{A}\cos\epsilon_1+\mathrm{B}\cos\epsilon_2 = 40, \qquad \mathrm{A}\sin\epsilon_1+\mathrm{B}\sin\epsilon_2 = 0.$

The values at Batavia suggest taking

$$3\mathrm{A}\cos\epsilon_1-4\mathrm{B}\cos\epsilon_2 = 80, \qquad 3\mathrm{A}\sin\epsilon_1-4\mathrm{B}\sin\epsilon_2 = 40.$$

Hence $\quad \mathrm{A} = 35, \qquad \mathrm{B} = 8, \qquad \epsilon_1 = 9°, \qquad \epsilon_2 = -45°.$

This again makes B about one-fourth of A.

But the harmonic was really made up of a main part of order 3 and a minor part of order 1, and the correction to V is thus found to be

$$\sin\theta\,\{-16\sec 45°\sin(\phi+15°)\cos 2t+16\sec 45°\cos(\phi+15°)\sin 2t\},$$

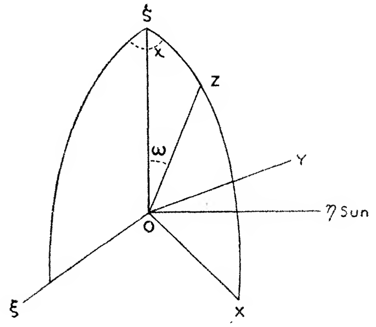
where we have assumed that the first order term was entirely external. The net result is

$$\begin{aligned}\mathrm{V} = {} & \{-120\sin^2\theta\cos\theta+80\sin^3\theta\sin(\phi-30°)-23\sin\theta\sin(\phi+15°)\}\cos 2t \\ & +\{40\sin^3\theta\sin(\phi-30°)+23\sin\theta\cos(\phi+15°)\}\sin 2t.\end{aligned}$$

The computed values in Table IV seem to give very satisfactory agreement with the data.

I do not, of course, suggest that this empirical representation of the data in Table I is unique or final, but I do think it is clear that the Schuster terms alone will not co-ordinate the data, and the form we have obtained does go a very considerable way towards meeting the difficulties.

We may now consider how the terms were arrived at, in such a way as to provide a clue to the physical meaning without binding oneself to any special theory.



In the figure let $O\xi$, $O\eta$, $O\zeta$, be rectangular axes through the earth's centre O, in the directions, earth's way, radius to the sun, and perpendicular to the plane of the ecliptic. Let OX, OY, OZ, be fixed rectangular axes in the earth, but not rotating with it, OZ being the axis of rotation, and OX in the plane containing $O\zeta$ and OZ. ω is thus the obliquity of the ecliptic and χ an angle expressing the time of the year.

The direction cosines of OX, OY, OZ, referred to $O\xi$, $O\eta$, $O\zeta$, are

$$\begin{aligned} OX &= \cos\omega\cos\chi, \quad \cos\omega\sin\chi, \quad -\sin\omega; \\ OY &= \quad -\sin\chi, \qquad \cos\chi, \qquad 0; \\ OZ &= \sin\omega\cos\chi, \quad \sin\omega\sin\chi, \quad \cos\omega. \end{aligned}$$

Let θ', ϕ' be the polar co-ordinates of a point fixed on the earth, and let θ, ϕ refer to any other point on the earth.

Suppose we start with a potential $\Omega = 1/r$ and differentiate along the direction θ', ϕ'. We get a new potential function

$$\begin{aligned} \Omega_1 &= \left(\sin\theta'\cos\phi'\frac{\partial}{\partial x}+\sin\theta'\sin\phi'\frac{\partial}{\partial y}+\cos\theta'\frac{\partial}{\partial z}\right)\frac{1}{r} \\ &= -\frac{1}{r^2}\{\sin\theta'\sin\theta\cos(\phi-\phi')+\cos\theta'\cos\theta\}. \end{aligned}$$

Since $\phi-\phi'$ remains constant while the earth rotates, we simply have a contribution to the permanent field of the earth, representing a doublet with its axis in the direction θ', ϕ'. Again, let us differentiate along $O\eta$. We get

$$\begin{aligned} \Omega_2 &= \frac{\partial}{\partial\eta}\frac{1}{r} = -\frac{\eta}{r^3} \\ &= -\frac{1}{r^2}\{\sin\theta\cos\phi\cos\omega\sin\chi+\sin\theta\sin\phi\cos\chi+\cos\theta\sin\omega\sin\chi\} \\ &= -\frac{1}{r^2}\{-(1-\sin^2\omega\sin^2\chi)^{\frac{1}{2}}\sin\theta\cos t+\cos\theta\sin\omega\sin\chi\}, \end{aligned}$$

where t is the local time at the point of observation reckoned from midnight.

The term in $\cos\theta$ contributes nothing to diurnal variation, but contributes a seasonal term to the general field. It has a maximum in summer, a minimum in winter, and vanishes at the equinoxes.

The term in $\sin\theta$ contributes to the diurnal variation. The amplitude has a semi-annual seasonal variation, rising to its maximum value 1 at the equinoxes, and falling to its minimum value $\cos\omega$ at the solstices.

Averaged for the year it contributes a term $\sin\theta\cos t$ to the mean diurnal variation.

Such a term is indicated by the data. We may, if we like, picture it as a uniform field in the direction of the sun, within which the earth rotates.

In a similar way we may differentiate along $O\xi$, and so get a term $\sin\theta\sin t$.

Let us now carry the process of forming functions further by differentiating Ω_1 along the direction $O\eta$. We may write the result in the form

$$\begin{aligned}\Omega_3 = \frac{1}{r^3}\{&3\sin\theta'\sin\omega\sin\chi\sin\theta\cos\theta\cos(\phi-\phi')-\cos\theta'\sin\omega\sin\chi(1-3\cos^2\theta)\\ &-3\cos\theta'\sin\theta\cos\theta(1-\sin^2\omega\sin^2\chi)^{\frac{1}{2}}\cos(\phi+t')\\ &-3\sin\theta'\sin^2\theta(1-\sin^2\omega\sin^2\chi)^{\frac{1}{2}}\cos(\phi+t')\cos(\phi-\phi')\\ &+\sin\theta'(1-\sin^2\omega\sin^2\chi)^{\frac{1}{2}}\cos(\phi'+t')\},\end{aligned}$$

where t' is Greenwich mean time, θ', ϕ' the co-ordinates of a point fixed with respect to Greenwich, and θ, ϕ the co-ordinates of the station referred to Greenwich.

The first two terms contribute nothing to diurnal variation, but only annual seasonal terms to the general field. The remaining terms contribute to diurnal variation, the amplitude having a semi-annual seasonal variation fluctuating between 1 and $\cos\omega$. Averaged for the year we get terms in the mean diurnal variation.

If θ' is 0 we have a term $\sin\theta\cos\theta\cos t$, where t is the local time. Similarly, a term $\sin\theta\cos\theta\sin t$ arises by differentiating along $O\xi$. We thus get the Schuster terms in the 24-hour term of diurnal variation.

If $\theta' = \frac{1}{2}\pi$, we have the term

$$3\sin^2\theta\cos(\phi+t')\cos(\phi-\phi')-\cos(\phi'+t'). \tag{A}$$

Similarly, differentiating along $O\xi$, we get a term

$$3\sin^2\theta\sin(\phi+t')\cos(\phi-\phi')-\sin(\phi'+t').$$

Now, in this latter form replace ϕ' by $\frac{1}{2}\pi+\phi'$, as we are at liberty to do. We get

$$3\sin^2\theta\sin(\phi+t'),\ \sin(\phi-\phi')-\cos(\phi'+t'). \tag{B}$$

Adding (A) and (B), we get

$$(3\sin^2\theta-2)\cos(\phi'+t'),$$

where t' is Greenwich mean time.

Again, deduct (B) from (A) and we get

$$3 \sin^2 \theta \cos (2\phi - \phi' + t').$$

These are the terms we found it desirable to introduce.

We have thus obtained the terms required for the data, and clearly we might have taken a more general form had it been necessary.

We have seen that Schuster proposed to explain his term as arising from the permanent field and tidal oscillation. But there are great difficulties in the way. Our analysis confirms the association with the permanent field, but otherwise may simply mean differential conductivity of the air as between midday and midnight, and as between sunrise and sunset, the difference arising from the ultra-violet radiation from the sun.

As regards the term depending on Greenwich mean time, its mode of generation would appear to associate it with a magnetic axis lying in the plane of the equator. If there is such an axis the intensity will have to be feeble if it is not to produce a very pronounced influence on the general field, as indicated by observations.

My main objective has been the mean diurnal variation for the year, letting the seasonal variation take care of itself. But I anticipated that any correct method of approach would provide by extension for the seasonal change.

This appears to be the case in the analysis given. Seasonal change is provided for, but the terms we have obtained contribute only semi-annual change, not annual change.

The latter is important, but will not of course leave an influence on the mean yearly value. Thus it may be expected to arise in a way which differs from that we have already used. But differentiation along $O\zeta$ supplies the kind of function wanted.

Thus $$\frac{\partial \Omega}{\partial \zeta} = -\frac{1}{r^2}\{\cos \omega \cos \theta - \sin \omega \sin \theta \sin (t - \psi)\},$$

where t is the local time and $\tan \psi = \cos \omega \tan \chi$.

The first term contributes to the permanent field.

The second term contributes to the diurnal variation at any particular time of the year. But the phase ψ passes through a cycle in the course of the year, and so contributes nothing when averaged for a year.

The seasonal change is, however, beyond the limits I had prescribed myself in this paper, and I shall not pursue it further on this occasion.

So far the examination has referred to the 24-hour terms, but the data indicate the necessity for adding to the Schuster expression $\sin^2 \theta \cos \theta \cos (2t + \alpha_2)$ in the 12-hour terms also. Our method for the

24-hour term suggests that we should get new 12-hour terms by a second differentiation along $O\xi$ or $O\eta$. The formulæ become more complicated if we are to retain the sun's annual variation in declination.

We therefore simplify the matter by regarding the sun as remaining in the plane of the equator. If we thus lose sight of the seasonal variation, we gain in conciseness.

If we take the form $\frac{\partial^2}{\partial\eta^2}\frac{1}{r}$, *i.e.* $\frac{\partial\Omega_2}{\partial\eta}$, we get as a potential form $-\frac{1-3\eta^2/r^2}{r^3}$ which leads to

$$-\frac{1}{r^3}\{(1-\tfrac{3}{2}\sin^2\theta)-\tfrac{3}{2}\sin^2\theta\cos 2t\}.$$

The contribution to the 12-hour term is of the form $\sin^2\theta\cos 2t$, but I did not find this term of much assistance in representing the data.

The Schuster term $\sin^2\theta\cos\theta$ arises by putting $\theta' = 0$ in Ω_1, which then becomes $\cos\theta/r^2$, and then forming the $\partial^2/\partial\eta^2$, $\partial^2/\partial\xi^2$, or $\partial^2/\partial\xi\partial\eta$ of $\cos\theta/r^2$.

If, on the other hand, we put $\theta' = \frac{1}{2}\pi$ in Ω_1, which then becomes $1/r^2 . \sin\theta\cos(\phi-\phi')$, and then form the $\partial^2/\partial\eta^2$, $\partial^2/\partial\xi^2$, or $\partial^2/\partial\xi\partial\eta$ of this, we arrive at 12-hour terms of the forms—

$$3\sin\theta\cos(\phi+\phi'+2t')-\tfrac{15}{2}\sin^3\theta\cos 2t\cos(\phi-\phi') \qquad \text{(C)}$$

and

$$3\sin\theta\sin(\phi+\phi'+2t')-\tfrac{15}{2}\sin^3\theta\sin 2t\cos(\phi-\phi').$$

In the latter replace ϕ' by $\frac{1}{2}\pi+\phi'$. We get

$$3\sin\theta\cos(\phi+\phi'+2t)-\tfrac{15}{2}\sin^3\theta\sin 2t\sin(\phi-\phi'). \qquad \text{(D)}$$

Addition of (C) and (D) gives the form

$$\sin\theta(\tfrac{4}{5}-\sin^2\theta)\cos(2t+\phi'-\phi),$$

while, if we subtract (D) from (C), we get the form

$$\sin^3\theta\cos(2t+\phi-\phi').$$

I found it convenient to add to these forms a term $\sin\theta\cos(2t+\phi'-\phi)$ which may be regarded as a uniform field parallel to the plane of the equator rotating with the angular velocity of the earth but in the opposite direction.

We have thus formed a simple specification of the terms which seem to fit with the data. I have no theory to propose as to their origin, beyond Schuster's view that they arise from differential conductivity of the upper regions of the atmosphere. If further data confirm the probable reality of the terms, it should not be a difficult matter to express the law of conductivity which would account for them. As we have only had to proceed to second differentials with regard to the sun's direction, or the earth's way, it does not look as if the differential conductivity required is of a very complex nature.

The possibility of associating the electric and magnetic state of the earth with its translational movement has often attracted attention, although without success. It may not be out of place here to give a general solution of the electromagnetic equations for a body of conductivity c moving in a straight line with velocity kC, where C is the velocity of radiation. The solution, which I believe is new, refers to the steady state.

If (X, Y, Z) (α, β, γ) are the components of electric force and magnetic force the equations are

$$\left(-k\frac{\partial}{\partial x}+c\right)(\mathrm{X}, \mathrm{Y}, \mathrm{Z}) = \left(\frac{\partial\gamma}{\partial y}-\frac{\partial\beta}{\partial z},\ \frac{\partial\alpha}{\partial z}-\frac{\partial\gamma}{\partial x},\ \frac{\partial\beta}{\partial x}-\frac{\partial\alpha}{\partial y}\right),$$

$$\left(-k\frac{\partial}{\partial x}\right)(\alpha, \beta, \gamma) = \left(\frac{\partial\mathrm{Y}}{\partial z}-\frac{\partial\mathrm{Z}}{\partial y},\ \frac{\partial\mathrm{Z}}{\partial x}-\frac{\partial\mathrm{X}}{\partial y},\ \frac{\partial\mathrm{X}}{\partial y}-\frac{\partial\mathrm{Y}}{\partial x}\right),$$

and

$$\frac{\partial\alpha}{\partial x}+\frac{\partial\beta}{\partial y}+\frac{\partial\gamma}{\partial z} = 0,$$

$$\left(-k\frac{\partial}{\partial x}+c\right)\left(\frac{\partial\mathrm{X}}{\partial x}+\frac{\partial\mathrm{Y}}{\partial y}+\frac{\partial\mathrm{Z}}{\partial z}\right) = 0.$$

A solution is expressed by

$$\alpha = -(1-k^2)\frac{\partial\psi_0}{\partial x}-kc\psi_0, \qquad \beta = -\frac{\partial\psi_0}{\partial y}, \qquad \gamma = -\frac{\partial\psi_0}{\partial z};$$

$$\mathrm{X} = 0, \qquad \mathrm{Y} = -k\frac{\partial\psi_0}{\partial z}, \qquad \mathrm{Z} = k\frac{\partial\psi_0}{\partial y},$$

where ψ_0 is a solution of

$$(1-k^2)\frac{\partial^2\psi_0}{\partial x^2}+kc\frac{\partial\psi_0}{\partial x}+\frac{\partial^2\psi_0}{\partial y^2}+\frac{\partial^2\psi_0}{\partial z^2} = 0.$$

Primary solution—

$$\psi_0 = \frac{1}{\rho}e^{-\frac{kc}{2(1-k^2)}x\pm\frac{kc}{2(1-k^2)^{1/2}}\rho},$$

where

$$\rho^2 = \frac{x^2}{1-k^2}+y^2+z^2,$$

and other solutions may be obtained by the usual process of differentiation.